Assignment 2 [10 points]

Consider the SIR model with demography with birth - and death rate . The SIR equations, where all symbols have their standard meaning, are

;

;

.

Consider now the case of paediatric vaccination, where a fraction of newborns are vaccinated and therefore protected from infection.

1. Adopt the standard SIR model from above to include this paediatric vaccination. (1 point)

,

,

.

By a change of variable, , , , and writing your model from (2) in terms of , , , you find that it has exactly the same form as the standard SIR equations, but with one important modification.

1. Proof that with this change of variables you can indeed bring the model with paediatric vaccination in exactly the same form as the standard SIR equations (2 points).

First, substitute into the model:

,

,

,

Second, simplify the third equation for :

,

,

,

Finally, divide all equations by :

,

,

,

QED

1. What is the major difference between the model from (2) and the standard SIR model? And what does this mean for the dynamics of the model? (2 points)

The most important difference is that if now scale by , so effectively making it smaller. This means for the dynamics that is reduced by a factor , so if p is large enough, can be pushed below 1, and making the disease go extinct.

1. The basic reproductive ratio for measles is 17. What fraction of the infant population should be vaccinated to prevent measles from spreading in the population. Explain why? (2 points)

, so, . This is based on assignment (4) where we found that is reduced by , so choosing such that is smaller than 1 has the desired effect, because in standard SIR with demography where is smaller than 1 always results in the disease going extinct, without having some endemic state.

Childhood diseases, for which paediatric vaccination is applied, are off course characterised by a strong seasonal forcing effect.

1. Mention at least three possible features of the SIR dynamics in the presence of seasonal forcing? (1point)

1) recurring yearly epidemics; 2) harmonic resonance; 3) subharmonic resonance, periodic doublings, transitions to chaos.

Finally, consider dynamic variability in childhood disease incidence in real data. The graph below is based on Figure 5.16 from Keeling and Rohani, Case reports for measles in London 1944 to 1988. The black line demonstrates weekly reported cases, with the gray line depicting the per capita birth rate. The dashed grey line demonstrates effective birth rate, correcting for vaccination, that started in 1968.



1. Describe the types of dynamics that you observe, and relate this back to what you know about SIR models with seasonal forcing, and what you discussed w.r.t. vaccination. (2 points)

Until 1950, we see harmonic resonance, an outbreak every year. From 1950 – 1968 there is subharmonic resonance, with an outbreak every two years. This is probably due to the lower birth rate that pushed the dynamics through a bifurcation leading to periodic doubling. From 1968 onwards we find more periodic doublings and a transition to chaotic dynamics. The vaccination pushes the effective birthrate way down, bringing the dynamics in this regime.

Assignment 3 [10 points]

1. What is stochastic extinction, and when is this most likely to happen? (2 points)

This is when in a stochatistic discrete event model by chance the amount of infected individuals drops to zero, making the disease go extinct (even when in a continuous model it would not do that). This most likely happens in small populations, diseases that undergo large amplitude oscillations (due to e.g. strong seasonal forcing), and diseases with small .

In an invading scenario, where a single infected individual enters a fully susceptible population, stochastic extinction can also occur.

1. What is meant with stochastic extinction in this scenario, and what is the probability of this actually happening (no need to derive of proof this)? (1 points)

Chance that an invading infected individual recovers before passing the infection to a secondary case. The chance for this to happen is .

Next assume additional periodic forcing, which will lead to periodic outbreaks in large enough populations.

1. What happens with the observed dynamics in smaller and smaller populations. You may assume a scenario including immigrants. Take into account the concept of Critical Community Size. (2 points)

In large populations we expect to see behaviour as in continuous models, recurring infections due to the seasonal forcing. As the population size goes down, the recurring infections are there, but with added noise, and for smaller populations the noise start to disrupt the nice continuous pattern. If the population size drops below the CCS, stochastic extinction can happens, so the infection dies out, and for years nothing may happen. However, due to immigrants, a new epidemic could be started, leading to isolated -in time- outbreaks (like in the data for Iceland shown during lectures).

1. What is a metapopulation model in infectious disease modelling? (1 point)

A model where you subdivide the population in distinct subpopulation (cities, counties, etc), each having independent dynamics, and some limited form of interaction.

Consider two subpopulations with a one-way coupling, meaning population 1 is coupled to population, but not vice-versa. Also assume that population 1 is large enough the be described as fully deterministic.

1. Assuming that population 2 is also deterministic, formulate the SIR equations with demography for these coupled populations. You may assume equal birth/date rates and recovery rates in both populations (1 point)

, ,

, ,

, .

For these two coupled deterministic models, now assume that an infection is introduced in population 1, and that population 2 is fully susceptible. Also assume that this infection has a basic reproductive ratio that allows it to spread and cause an epidemic

1. Explain in words the dynamics in this coupled model. (1 point)

Population 1 will develop a standard epidemic, and then settle into the endemic state. In population 2 the epidemic will also immediately develop, due to the deterministic coupling. Moreover, analysis shows, as demonstrated in the lectures, that the early dynamics in population 2 is exponential growth with a rate . Since the dynamics in 2 in coupled to those of 1, it is hard to say without simulating how the infection in 2 would develop.

Finally assume that population 2 is too small to be considered fully deterministic. Replace the model population 2 now with a stochastic SIR model.

1. What do you observe now? Take the strength of coupling between population 1 and 2 into account in your discussion (2 points)
   1. If the coupling is small enough, there is a high probability that the disease will not spread in population 2
   2. For larger coupling the pathogen can spread, but their can be a significant delay of spreading of the disease in population 2 with respect to the epidemic in population 1